Case Study: Forecasting of a Time Series Signal using GARCH Techniques.

Introduction:

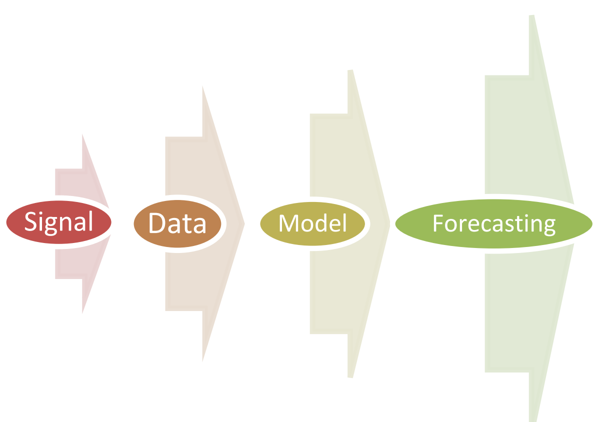
Forecasting is a process of predicting or estimating the future based on past and present data. Time series analysis method is used for analyzing time series data in order to extract meaningful statistics and other characteristics of the data. Time series forecasting is the use of a model to predict future values based on historical data.

GARCH Model:

GARCH model is a Generalised Auto regressive Conditional Heteroskedasticity that is an extension of the ARCH model, If a series exhibits volatility clustering; this suggests that past variances might be predictive of the current variance. Here changes in variability are related to, or predicted by, recent past values of the observed series.

This case study deals with the forecasting of already featured time series data using a GARCH Model.

Steps of Implementation:



Signal Generation:

A total 20 input data signals S100 to S119 are generated at 8Hz, 8.5Hz, 9Hz, 9.5Hz, 10Hz, 10.5Hz, 11Hz, 11.5Hz, 12Hz, 12.5Hz, 13Hz, 13.5Hz, 14Hz, 14.5Hz, 15Hz, 15.5Hz, 16Hz, 16.5Hz, 17Hz and 17.5Hz. The total time period of 2 seconds is divided into 200 samples by sampling the signal at a rate of 0.01 seconds. The signals S100 to S119 generate **4000** samples.

$$\text{Total number of samples} = \frac{T}{R} \times n$$

T = Total time period = 2 seconds

R = Sampling rate = 0.01 seconds

n = Number of input signals = 20

Data Acquisition:

Set is collected from the input signal as the signal is varying continuously. It has different value of amplitude at different interval of time. The data is collected for all the different input signals generated.

Model:

The generalized autoregressive conditional heteroskedastic (GARCH) model is an extension of Engle's ARCH model for variance heteroskedasticity.

In an ARCH model, next period's variance only depends on last period's squared residual so a crisis that caused a large residual would not have the sort of persistence that we observe After actual crises. The GARCH model is described below,

$$\boldsymbol{\delta^2_t = \alpha_0 + \alpha_1\, \alpha^2_{t-1} + \beta_1\, \delta^2_{t-1}}$$

Where $\alpha_0 > 0$, $\alpha_1 > 0$, $\beta_1 > 0$, and $\alpha_1 + \beta_1 < 1$, so that our next period forecast of variance is a blend of our last period forecast and last period's squared return.

Forecasting:

The generated data samples are considered as input to the GARCH model and assigned the time period to forecast the future data signal.

Methodology:

There are many different types and flavors the simplest one being GARCH(1,1) having
S[n+1] = S[n] + S[n] mu + sqrt{v[n]} B[n]
the time series is a discrete sequence in the real world with mu the real world drift and possibly time-dependent and B[n] sequence of independent Brownian Motions v[n] the variance. It Is well established, that the autocorrelation and partial autocorrelation functions are useful tools in identifying and checking time series behavior of the ARMA Form in the conditional mean. Similarly The autocorrelations and partial autocorrelations for the squared proces prove helpful in identifying and checking GARCH Behavior in the conditional variance equation. The future values are forecasted by the GARCH by varying the above mentioned parameters sequentially.

Result:

The forecasted future data signal for 5, 8, 15 and 20 seconds is generated for time varying signal of 8Hz. The input signal and predicted signal waveforms are shown in figures.

Seconds	Value	Seconds	Value
1	0.4814	14	0.7029
2	0.5955	15	0.7029
3	0.6481	16	0.7029
4	0.6743	17	0.7029
5	0.6875	18	0.7029
6	0.6949	19	0.7029
7	0.6986	20	0.7029
8	0.7006		
9	0.7017		
10	0.7022		
11	0.7025		
12	0.7027		
13	0.7028		

Conclusion:

The ARCH/GARCH is very successful in predicting volatility changes. As we have seen, ARCH/GARCH models describe the time evolution of the average size of squared errors, that is, the evolution of the magnitude of uncertainty. Despite the empirical success of ARCH/GARCH models, there is no real consensus on the economic reasons why uncertainty tends to cluster. That is why models tend to perform better in some periods and worse in other periods.
It is relatively easy to induce ARCH behaviour in simulated systems by making appropriate assumptions on agent behaviour.